# Preface

This special volume of the Journal ESAIM: COCV (European Series in Applied and Industrial Mathematics: Control, Optimisation and the Calculus of Variations) is dedicated to the memory of Jacques-Louis Lions who paved the way to contemporary Applied Mathematics, and who profoundly influenced many researchers in the areas of Control, Optimisation, and the Calculus of Variations, among others. As a tribute to Lions' pivotal legacy, the Editorial Board of ESAIM: COCV has invited some of his former students and collaborators to contribute towards a special issue in his memory. For their help with this venture, we are grateful to the group of former students of Lions who undertake the organization of the Conference dedicated to his memory, and specially to François Murat and Jean-Pierre Puel who, together with Jean Cea, were at the root of the foundation of ESAIM.

ESAIM: COCV is a product of Lions' scientific vision. Indeed, the project of launching a series of new collections of journals and proceedings under the name ESAIM came to fruition in 1994 under the auspices of the French Ministry of Education and due in part to the encouragement and support of Lions. Lions' commitment to ESAIM was well attested by the fact that he published the first paper of ESAIM: COCV, at the time when the journal was only available electronically.

Lions was a truly applied mathematician who very soon realized the interest and impact of treating problems motivated and relevant to the industrial and technological development. Furthermore, Lions advocated a mathematics research with no geographical boundaries and practiced in a truly international setting.

This volume contains a number of research papers as well as a a biography of Jacques-Louis Lions that Philippe G. Ciarlet wrote last winter for the EMS-newsletter. We thank Philippe and the EMS-newsletter for granting us permission to include his article in this volume. While Ciarlet's article describes briefly, but thoroughly, Lions' major mathematical contributions, this is not our aim in this short introduction. Nonetheless, we would like to close with a reference to some of Lions' mathematical highlights.

After a period of intensive research on the foundations of the modern theory of PDE and Numerical Analysis, Lions published in 1969 his celebrated book on Nonlinear Partial Differential Equations, which is still a landmark and a reference in this field. Later he turned his attention to Homogeneization problems, *i.e.* equations or systems with highly oscillating coefficients. His monography, written in collaboration with Alain Bensoussan and George Papanicolaou, surveys a variety of elliptic, parabolic, and hyperbolic problems and solves them within the framework of a unified vision.

We now move on to topics which are more closely related to the Journal ESAIM: COCV. In 1968 Lions published a book in French on the optimal control of systems governed by partial differential equations, where he generalized to systems of partial differential equations the theory of Pontryagin and co-authors which dealt with the control of systems governed by ordinary differential equations, and which constituted one of the major progresses in the mathematical Control Theory at the time. He studied in a extensive way the control of elliptic, parabolic and hyperbolic systems, both for open loop and closed loop (feedback) control. This book was translated into English in 1971 and had a great

influence in the field. Several students of Lions investigated new aspects of the theory, such as the control of variational inequalities, and some others developed the numerical approximation methods for these problems, an issue of primary importance in Control Theory.

Lions was also very interested in the practical applications of this newborn mathematical theory in various domains of physics, mechanics or industry, such as the control of flexible structures, of industrial furnaces, biochemical processes, controlled nuclear fusion, etc., and in this he was truly a pioneer. He encouraged several of his students, at INRIA, Paris 6, and at other research institutions, to work on this type of applied problems. This research avenue soon became very successful and Lions was very proud of it.

Later Lions published a book on the control of distributed singular systems, where he studied the control of unstable systems or of systems with multiple states and he derived the Singular Optimality System. Lions entertained an intense collaboration with A. Bensoussan on stochastic control theory and published two books with him on this topic, one of them in the Wiley/Gauthier-Villars Series in Modern Applied Mathematics that Lions launched as editor together with J. Dixmier. He published the so-called "Chinese book" on "Some methods in the mathematical analysis of systems and their control" with the contribution of Li-Ta-Tsien, who, in turn, wrote several appendices and translated it into Chinese for the Science Press in China. The yearly lectures of Lions at the Collège de France during his tenure of the chair "Analyse et Contrôle des Systèmes" since 1973, where he systematically addressed a new topic every academic year, were an internationally recognized reference and soon became a guide for research efforts in the field. Those courses were an invaluable source of open problems that motivated young French researchers, and attracted to Paris many foreign scientists.

Later Lions became interested in controllability problems, where one is concerned with the question of whether a given evolution system may be driven from any initial state to any final state in a given finite time. In 1986 Lions published two Notes in the CRAS where he introduced the so-called, and by now celebrated, HUM (Hilbert Uniqueness Method) for the controllability of linear evolution equations. Lions developed his theory in more detail in his John von Neumann lecture published by SIAM Reviews, in the occasion of his nomination as Prize Winner. This topic became the theme of his courses at the Collège de France in the academic years 1987-1988 and 1988-1989, and influenced many others to a large extent. Both courses were published in the RMA collection of Masson in 1988. In the first volume (Tome 1, published in Volume 8 of that collection) Lions offers a systematic analysis of the foundations of his theory and explores its applications to the controllability of systems for vibrations (wave, plate and elasticity models, mainly) by means of multiplier techniques, which allow him to establish explicit inequalities for the norms defined through the adjoint system. He also introduces notions that are by now classical such as partial and simultaneous controllability, and which play a central role in the controllability of large systems. In the second volume (Tome 2, published in Volume 9 of RMA) Lions explores the controllability of irreversible systems and analyzes the robustness of this property under a large variety of perturbations of the system under consideration: homogenization, singular perturbations, etc. A third book in the RMA collection, in collaboration with J. Lagnese, is dedicated to the modeling and analysis of stability of vibrating plates. The problem of stabilization, very closely connected to that of controllability, consists in finding convenient damping feedback mechanisms to guarantee the stability of a given system. These two problems are not only close to each other from a control theoretical point of view, but also they may be handled by means of similar methods relying on the use of multipliers to derive new energy estimates.

These works constitute a major contribution of Lions and have been the starting point of a fruitful period of research in this area for the last 15 years. Some of the developments in this field are summarized in this volume. Most of Lions' later contributions, often in different areas such as modeling

in climatology, numerical methods, domain decomposition techniques, etc., has been influenced and inspired by this methodology, and these type of problems remained one of his major scientific concerns until his very last days.

There are still many important open problems to be solved in Control Theory, a field which is clearly divided into the era before and after Lions' contributions in the second half of the 80's. The influence of Lions is not only due to his technical and methodological contributions, but also to his development of a new concept of controllability: all mathematical methods and results are good if they may be interpreted in physical terms and may have potential applications and, conversely, all issues arising in control applications have their counterpart in the context of controllability of mathematical systems which deserve a mathematical analysis.

The premature loss of Jacques-Louis Lions in Spring 2001 has saddened the mathematics community and created a void that will remain open in years to come. A great mathematician, master and friend is gone, and with him an exciting era of scientific discoveries. With this collection of works we aim to express the gratitude of the mathematics community in the areas of the Calculus of Variations, Control and Optimization to Lions for his unforgettable and pivotal contributions, both from the mathematical and human points of view. Given the immense far-reaching influence of Lions, it will be impossible to include in this volume contributions from all leading and very distinguished mathematicians who have benefited from Lions' scientific wisdom in one way or another, and here rest our apologies to all those who have not been contemplated.

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