ERRATUM OF “THE SQUARES OF THE LAPLACIAN-DIRICHLET EIGENFUNCTIONS ARE GENERICALLY LINEARLY INDEPENDENT”

Yannick Privat\textsuperscript{1} and Mario Sigalotti\textsuperscript{1,2}

Received September 2nd, 2009.
Published online December 4, 2009.

ESAIM: COCV 16 (2009) 794–805

Firstly, we would like to clarify that domains are, by definition, connected. This is not precisely stated in the definition of $\Sigma_m$ in Section 2.1.

Secondly, in the proof of Theorem 2.4 we claim that “each $\Lambda_k(t)$ converges, as $t \to +\infty$, to an eigenvalue of the Laplacian-Dirichlet operator on $\tilde{\Omega}$”. In general, this is not true.

The result stated in Theorem 2.4, however, is true and can actually be strengthened as follows.

**Theorem 2.4.** Let $(F_n)_{n \in \mathbb{N}}, (P_n)_{n \in \mathbb{N}}$ and $(R_n)_{n \in \mathbb{N}}$ be as in the statement of Theorem 2.3. Then, for every $m \in \mathbb{N} \cup \{+\infty\}$, a generic $\Omega \in \Sigma_m$ satisfies $P_n$ for every $n \in \mathbb{N}$.

A proof of this result can be based on the following strengthened version of Proposition 2.2.

**Proposition 2.2 (Teytel).** Let $m > 2$ and $\Omega_0, \Omega_1$ be two domains in $\Sigma_m$ that are $C^m$-differentially isotopic. Then there exists an analytic curve $[0, 1] \ni t \mapsto Q_t$ of $C^m$-diffeomorphisms such that $Q_0$ is equal to the identity, $Q_1(\Omega_0) = \Omega_1$ and the Laplacian-Dirichlet operator has simple spectrum on every domain $\Omega_t = Q_t(\Omega_0)$ for $t$ in the open interval $(0, 1)$.

Teytel proves Proposition 2.2 in the case where $\Omega_0$ and $\Omega_1$ are $C^m$-differentially isotopic to the unit $d$-dimensional ball. His argument applies also, without modifications, to pairs of domains belonging to the same isotopy class.

The proof of Theorem 2.4, in its new formulation, works by replacing:

- “Fix $m \in \mathbb{N} \cup \{+\infty\}$. [...] We are left to prove that $\tilde{A}_j$ is dense in $\Sigma_m$.” with “Fix $m \in \mathbb{N} \cup \{+\infty\}$.

Thanks to Theorem 2.3, a generic $\Omega \in D_m$ satisfies $P_n$ for every $n \in \mathbb{N}$. Fix one such $\Omega$ and notice that, in particular, the spectrum $(\lambda_n^\Omega)_{n \in \mathbb{N}}$ is simple.

Define, for every $n \in \mathbb{N}$, the set

$$\tilde{A}_n = \{\Omega \in \Sigma_m \mid \Omega \text{ satisfies } P_n\}.$$

The openness of $\tilde{A}_n$ in $\Sigma_m$ can be proved following exactly the same argument used in the proof of Theorem 2.3 to show that each $A_n$ is open in $D_m$. We are left to prove that $\tilde{A}_n$ is dense in $\Sigma_m$. Without loss of generality $m > 2$.”

\textsuperscript{1} Institut Élie Cartan de Nancy, UMR 7502 Nancy-Université – INRIA – CNRS, B.P. 239, 54506 Vandœuvre-lès-Nancy Cedex, France.

\textsuperscript{2} INRIA Nancy – Grand Est, France. Mario.sigalotti@inria.fr
Moreover, \( t \mapsto \Omega_t \) is an analytic path in \( \Sigma_m \). [...] Since, for \( t \) small enough, \( \Lambda_k(t) = \lambda_{j_k}^{\Omega_t} \), we deduce that \( \Omega \) can be approximated arbitrarily well in \( \Sigma_m \) by an element of \( \hat{A}_j \),” with “Moreover, each \( \Omega_t \) is isotopic to \( \Omega \). It follows from Proposition 2.1 that we can fix \( t \) large enough in such a way that \( \Omega_t \) verifies \( P_n \). Proposition 2.2 implies that there exists an analytic path of domains \( s \mapsto \tilde{\Omega}_s \) such that \( \tilde{\Omega}_0 = \Omega \), \( \tilde{\Omega}_1 = \Omega_t \) and the spectrum of the Laplacian-Dirichlet operator on \( \tilde{\Omega}_s \) is simple for every \( s \in (0, 1) \).

Hence, as in the proof of Theorem 2.3, we can deduce that \( \tilde{\Omega}_s \) satisfies \( P_n \) for all but finitely many \( s \in [0, 1] \). In particular, \( \Omega \) is in the closure of \( \hat{A}_i \).”