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PREFACE

Special issue in honor of Jean-Michel Coron for his 60th birthday

In honor of Jean-Michel Coron, for his 60th birthday, we are pleased to introduce this special volume of "ESAIM: Control Optimization and Calculus of Variations".

Jean-Michel Coron has done fundamental and original contributions to variational methods for partial differential equations and to control theory, for nonlinear systems in finite dimension and for nonlinear partial differential equations. Some of his ideas have truly revolutioned the field. His impact is immense and Jean-Michel continues to amaze us with striking mathematical discoveries.

To celebrate his 60th birthday, some of his collaborators and former students have been invited to contribute to a special issue. We are grateful to all authors and referees of this volume, for their enthusiastic participation in this adventure.

The journal "ESAIM: Control Optimization and Calcul of Variations", funded by Jean-Michel Coron and Jean-Pierre Puel in 1995, and which has been directed by Jean-Michel from 1995 to 2002, was obviously the best journal to carry out this initiative. We thank the Editorial Board for granting us the permission to create this special volume and the whole staff of the journal for their kind help in this task.

This volume contains a number of research papers by great authors. We have a particular thought for Abbas Bahri, who passed away in January 2016. In spite of difficult circumstances, he wanted to honor his collaboration with Jean-Michel by contributing to this special issue. We received his article in October 2015 and we decided to publish it posthumous.

To celebrate Jean-Michel Coron 60th birthday, an international conference entitled "Nonlinear Partial Differential Equations and Applications" took place at Institut Henri Poincaré, in Paris, June 20–24th, 2016. During five days, academic scholars who all have intersected Jean-Michel's trajectory presented exciting and high-quality lectures, related to his contributions. The workshop was a great success: captivating speakers, large and fascinated audience, friendly atmosphere. It demonstrated the extended breadth of Jean-Michel's research interests, as well as the incredible impact of his mathematical contributions. The very large number of participants is a clear asset of Jean-Michel Coron's popularity in the scientific community.

Happy birthday Jean-Michel!

Karine Beauchard and Emmanuel Trélat

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Jean-Michel Coron's contributions

Jean-Michel Coron has had a deep and major impact in the study of variational methods for nonlinear partial differential equations. His original works on functionals with critical Sobolev exponents has become classical. These functionals appear in many geometrical problems, for example surfaces of prescribed mean curvature, harmonic maps from Riemann surfaces to any Riemannian manifold and scalar curvature issues. They do not satisfy the Palais–Smale conditions: there are sequences of almost critical points with a bounded energy which goes to "infinity". However these sequences go to infinity in a precise manner. Jean-Michel, in collaboration with Brezis, gave in [8] the most precise description of this manner. In particular he proved that these sequences concentrate at a finite number of points and that, if they concentrate at the same points, the speed of concentrations must be very different. Thanks to this precise behavior of these sequences, he showed in the importance of the topology of the domain for the existence of critical points. In particular, in [14], he proves that if a bounded domain Ω in $\mathbb{R}^3$ has a small hole, then the equation with a critical Sobolev exponent

$$-\Delta u = u^5 \text{ in } \Omega,\ u > 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega, \tag{1}$$

has a solution. This article shows in an impressive way the core of these type of equations and numerous extensions and generalizations have been published. In his striking article [2], in collaboration with Bahri, Jean-Michel succeeded to remove the smallness assumption on the hole. It has deeply influenced many researchers.

In [7], in collaboration with Brezis, Jean-Michel solved the Rellich conjecture: given a Jordan curve Γ in a ball of radius R in $\mathbb{R}^3$ and a real constant H different from 0 and such that $|HR| < 1$, there are at least two geometrically distinct surfaces of constant mean curvature H spanning Γ. The existence of a first surface (the "small" solution) was already known; he proved the existence of the second surface (the "large" one). Again these surfaces are critical points of functionals with a critical Sobolev exponent and the study of the lack of the Palais Smale condition is important. In particular this study shows that the lack of the Palais–Smale condition does not hold below a precise threshold. In the spirit of works by Aubin [1] and Brezis and Nirenberg [10], the key step is then to prove that a suitable topological argument (here the mountain pass lemma) leads to a potential critical value which is below this threshold: the potential critical value is then a true critical value.

Jean-Michel Coron did also seminal and fundamental works on harmonic maps and nematic liquid crystals. Harmonic maps are critical points of the Dirichlet energy $E(\varphi) := \int_M |\nabla\varphi|^2 \mathrm{d}x$ for maps $\varphi : M \to N$ where M and N are two Riemannian manifolds. If M is of dimension 2 we are again in the case of a critical Sobolev exponent. In [9], in collaboration with Brezis and E. Lieb, Jean-Michel considered the case where M is a domain in $\mathbb{R}^3$ and N is the unit sphere $\mathbb{S}^2$ of $\mathbb{R}^3$. In that case, one is "above" the critical exponent and E is a special case of the Oseen–Frank energy functional used in the theory for nematic liquid crystals. Jean-Michel gave sharp lower bounds on $E(\varphi)$ in terms of the locations of the singularities of φ and the degree of these singularities. He also proved that $g(x/|x|)$ is a local minimizer of E if and only if $\pm g$ is a rotation. This article has been the source of inspiration for many further researches. Later on Jean-Michel studied the heat flow associated to the functional E (still in the case where M is a domain of $\mathbb{R}^3$ and $N = \mathbb{S}^2$). The existence of a (natural) weak solution to the Cauchy problem was already known. In [15], he proved the nonuniqueness of the weak solution and in [28], in collaboration with J.-M. Ghidaglia, he proved that there are smooth initial data leading to blow-up in finite time.

At the beginning of the nineties Jean-Michel Coron moved to control theory. A control system is a dynamical system on which one can act by using suitable *controls*. Very often it is modeled by means of a differential equation of the following type

$$\dot{y} = f(y, u). \tag{2}$$

The variable y is the state and belongs to some space $\mathcal{Y}$. The variable u is the control and belongs to some space $\mathcal{U}$. The spaces $\mathcal{Y}$ and $\mathcal{U}$ can be of infinite dimension and the differential equation (2) can be a partial differential equation.

There are a lot of problems that appear when studying a control system. Among the most important ones there are the *controllability* problem and the *stabilization* problem.

Controllability. The controllability problem is, roughly speaking, the following one. Let us give two states. Is it possible to steer the control system from the first one to the second one by using a suitable control $t \mapsto u(t)$? In the framework of (2), this means that, given the state $a \in \mathcal{Y}$ and the state $b \in \mathcal{Y}$, does there exit a map $u : [0, T] \to \mathcal{U}$ such that the solution of the Cauchy problem $\dot{y} = f(y, u(t))$, $y(0) = a$, satisfies $y(T) = b$? If the answer is yes whatever are the given states, the control system is said to be globally controllable. If $T > 0$ can also be arbitrary small one speaks of small-time global controllability. If the two given states and the control are restricted to be close to an equilibrium one speaks of local controllability at this equilibrium. (An equilibrium of the control system is a point $(y_e, u_e) \in \mathcal{Y} \times \mathcal{U}$ such that $f(y_e, u_e) = 0$.) If, moreover, the time T is small, one speaks of small-time local controllability.

Stabilization. The stabilization problem can be understood with the classical experiment of an upturned broomstick on the tip of one's finger. In principle if the broomstick is vertical with a vanishing speed, it should remain at the vertical (with a vanishing speed). As one sees experimentally, this is not the case in practice: if we do nothing the broomstick is going to fall down. This is because the equilibrium is unstable. In order to avoid the fall, one moves the finger in a suitable way in order to stabilize this unstable equilibrium. This motion of the finger is a feedback: it depends on the position (and the speed) of the broomstick. One can see the usefulness of these informations (position and speed) by trying to do the experiment with closed eyes: it is much more difficult (and dangerous). Feedback laws are now used in many industries and even in everyday life (*e.g.* thermostatic faucets).

The problem of the stabilization is the existence and construction of such stabilizing feedback laws for a given control system. More precisely, let us consider the control system (2) and let us assume that $f(0, 0) = 0$. The (asymptotic) stabilization problem is to find a feedback law $y \mapsto u(y)$ such that 0 is asymptotically stable for the closed loop system $\dot{y} = f(y, u(y))$.

In many situations, only part of the state-called the output and denoted $z := h(y) \in \mathbb{R}^p$ later on-is measured and therefore state feedback laws cannot be implemented; only output feedback laws are allowed. Roughly speaking a control system is said to be observable if one can recover the state by applying suitable controls and by looking only at the output during some interval of time. In order to stabilize a control system by means of output feedback laws, it is important to consider what is called *dynamic* output feedback: one allows the feedback to depend on some extra variable $\tilde{y}$ whose dynamics can be chosen as one wants: the closed loop system has now the form

$$\dot{y} = f(y, u(h(y), \tilde{y})),\ \dot{\tilde{y}} = v(h(y), \tilde{y}), \tag{3}$$

where the dynamic feedback law is $(z, \tilde{y}) \in \mathbb{R}^p \times \mathbb{R}^k \mapsto (u(z, \tilde{y}), v(z, \tilde{y})) \in \mathbb{R}^m \times \mathbb{R}^k$. One is now looking for the maps u and v (and the dimension k of $\tilde{y}$) such that $(0, 0) \in \mathbb{R}^n \times \mathbb{R}^k$ is asymptotically stable for the closed loop system (3).

Let us summarize briefly some of the main results obtained by Jean-Michel Coron on these two issues.

Stabilization. As shown by Brockett in [11], many important nonlinear controllable systems cannot be stabilized by means of *stationary* (*i.e.* independent of time) feedback laws $y \mapsto u(y)$. In major pieces of work [16,20] Jean-Michel proved that (most of) the controllable systems can be (asymptotically) stabilized by means of periodic *time-varying* feedback laws $(t, y) \to u(t, y)$. In both articles an important part of the proof is that for controllable systems the linearized control systems are controllable for generic feedback laws. Jean-Michel developed this new result and provided new applications to the stabilization problem in [18]. Similarly, many important nonlinear observable and controllable systems cannot be stabilized by means of *stationary* (*i.e.* independent of time) dynamic output feedback laws, even if, as shown by Jean-Michel in [19], they are stabilizable by means of stationary feedback laws. In [19] Jean-Michel proved that (most of) the controllable and observable systems can be stabilized by means of periodic *time-varying* dynamic output feedback laws $(z, \tilde{y}, t) \to (u(z, \tilde{y}, t), u(z, \tilde{y}, t))$.

In order to construct explicit stabilizing feedback laws a strategy is to use control Lyapunov functions, *i.e.* functions that are minimal at the desired equilibrium and that one can make decrease thanks to a well chosen feedback law. In collaboration with B. d'Andréa-Novel and G. Bastin, Jean-Michel used this method to stabilize hyperbolic systems; see in particular [25], where he gave the weakest known sufficient condition for exponential stability for general nonlinear 1-D hyperbolic systems. These systems have the following form

$$y_t + A(y)y_x = 0,\ y \in \mathbb{R}^n,\ x \in [0, L],\ t \in [0, +\infty), \tag{4}$$

with $A(0) = \operatorname{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$, Just to simplify the notations, let us assume all the λ_i's are positive. The boundary condition considered can then be written in the following form

$$y(0) = G(y(L)), \tag{5}$$

where $G : \mathbb{R}^n \to \mathbb{R}^n$ is assumed to vanish at 0. Part of G is fixed, part of G can be chosen (this is the feedback law). Let $K := G'(0)$ and let

$$\rho_2(K) := \operatorname{Inf} \{\|\Delta K \Delta^{-1}\|;\ \Delta \in \mathcal{D}_{n,+}\}, \tag{6}$$

where $\mathcal{D}_{n,+}$ denotes the set of $n \times n$ real diagonal matrices with positive diagonal elements. In [25] Jean-Michel proved that, if $\rho_2(K) < 0$, then 0 is exponentially stable in all the Sobolev spaces $H^k(0, L)$ with $k \geqslant 2$ for the system (4)–(5). This has allowed d'Andréa-Novel, Bastin and Jean-Michel to construct control laws which are presently applied to regulate the rivers La Sambre and La Meuse in Belgium.

Related to this problem, in [31], Jean-Michel, in collaboration with Nguyen, proved that there are 1-D hyperbolic systems (4)–(5) for which the linearized system is exponentially stable in every natural functional space (C^k, H^l, *etc.*), the nonlinear system is exponentially stable in all Sobolev spaces $H^k(0, L)$ with $k \geqslant 2$ but is unstable in $C^1([0, L])$. It is a very surprising result.

Let us point out that in many cases it is important to have *rapid* stabilization. In cite [29], in collaboration with Lü, Jean-Michel introduced in the framework of a control system modeled by means of a Korteweg-de Vries equation a strategy to get it in some cases. It consists in looking for linear transformation allowing to transform the initial system into a target system for which the rapid stabilization is easy to get. This strategy is directly inspired from the backstepping approach initiated by Krstic and his school [39] where the linear transform is a Volterra transformation of the second kind. However, for this control system it is not clear if a Volterra transformation of the second kind is sufficient. Jean-Michel recently applied his strategy to other partial differential equations (Kuramoto–Sivashinsky equations [30] and Schrödinger equations).

Controllability. On this subject, Jean-Michel considered control systems modeled by means of partial differential equations. One knows many tools to study the controllability of such control systems if they are linear. Let us mention in particular (together with the articles where they have been introduced)

- harmonic analysis and Ingham's inequalities [45];
- multipliers method [36, 41, 42];
- microlocal analysis [3];
- Carleman's inequalities [33, 37, 38, 40].

From the controllability of the linearized control systems at some equilibrium one can often deduce the local controllability around this equilibrium by using a suitable inverse mapping theorem. Jean-Michel looked at cases where the linearized control system is not controllable. In finite dimension, there are many tools to deal with these cases, relying on iterated Lie brackets. In many important situations these tools do not work for control systems modeled by partial differential equations. Jean-Michel proposed various methods to treat these situations, in particular the return method, quasi-static deformations and power series expansion.

The return method. Jean-Michel's idea is the following one. One wants to study the local controllability around some equilibrium. The first thing to try is to linearize the control system around this equilibrium. If this linearized control system is controllable, the inverse mapping theorem allows to get that the non linear control

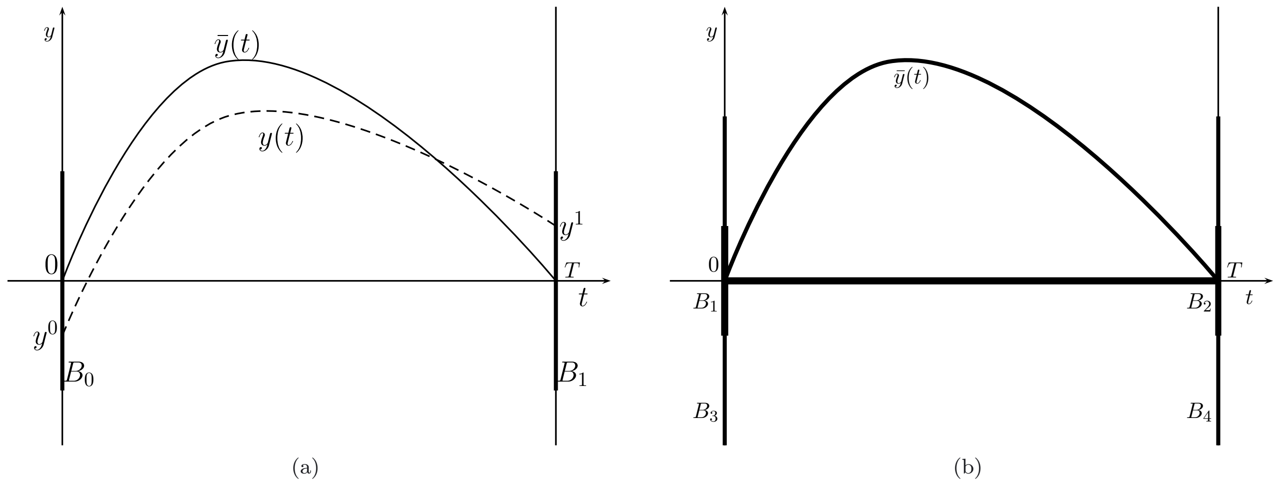

FIGURE 1. Return method: local and global controllability. (a) Return method and local controllability: the local controllability around 0 is deduced from the controllability of the linearized control system around the trajectory $\bar{y}$. (b) Return method and global controllability: the controllability around $\bar{y}$ is better than the controllability around 0: B_3 and B_4 are larger than B_1 and B_2.

system is locally controllable always in finite dimension and often in infinite dimension (but this last case is sometimes more difficult to handle due to problems of "loss of derivatives"). However if this linearized control system is not controllable, nothing can be deduced on the controllability of the nonlinear system. The idea is to look for trajectories going from the equilibrium and returning to the equilibrium such that the linearized controls system around these trajectories are controllable. If such trajectories exist one can get the local controllability of the nonlinear system. See Figure 1a. This is the funding principle of the famous "Coron's return method". Jean-Michel first introduced this method in [16] in order to stabilize finite dimensional control systems. In the framework of partial differential equations he introduced it in [17,22] to prove the global boundary controllability (in arbitrary small time) of the Euler equations of incompressible fluids in dimension 2. In this case, the control is the velocity field on Γ, which is an open subset of the boundary of the domain, as well as the vorticity of the incoming flow on Γ. He proved that this control system is globally controllable in small time if and only if Γ intersects every connected component of the boundary of the domain. This result is widely hailed as one of the most original results on the controllability of nonlinear partial differential equations. The case of the Euler equations of incompressible fluids in dimension 3 was treated later on by Glass in [34] using also the return method.

Even if the linearized control system around the equilibrium is controllable, Jean-Michel pointed out that it is still interesting in some cases to consider other trajectories going from the equilibrium and returning to the equilibrium and such that the linearized controls system around these trajectories are controllable. Indeed, when applying in a quantitative manner the inverse mapping theorem, they may lead to a "better" local controllability result and eventually may lead to global controllability. In Figure 1b, B_1 and B_2 are two neighborhoods of 0 such that every point in B_2 can be reached from every point of B_1 by following a trajectory of the control system which is close to 0; B_3 and B_4 are two neighborhoods of 0 such that every point in B_4 can be reached from every point of B_3 by following a trajectory of the control system which is close to the trajectory $\bar{y}$ which starts from 0 and return to 0. The size of the B_i's are estimated thanks to quantitative forms of the inverse mapping theorem. The interest of $\bar{y}$ compared to the null trajectory is that B_3 is larger than B_1 and B_4 is larger than B_2. Using this strategy Jean-Michel established impressive global controllability results for the Navier–Stokes equations in [21, 27].

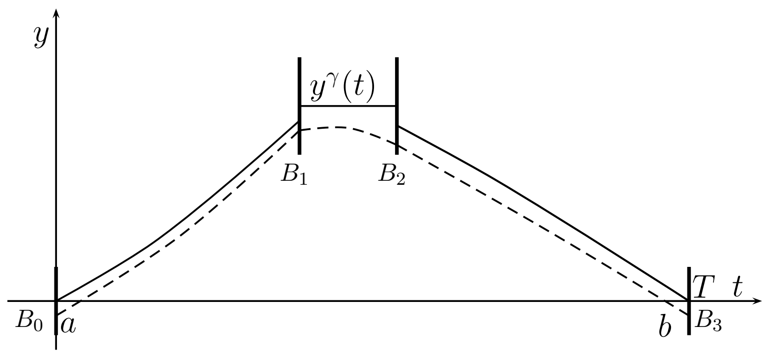

FIGURE 2. Quasi-static deformations. The linearized control system around y^γ, the equilibrium corresponding to the control $u = \gamma$, is controllable. Hence it is possible to move from every point in B_1 to every point in B_2. One moves from 0 to a point in B_1 by using controls changing very slowly (quasi-static deformations; continuous curve going from 0 to B_1). Similarly going backward in time and using controls changing very slowly one can reach 0 by starting from a suitable point in B_2 (continuous curve going from B_2 to 0). Hence one gets the existence of B_0 and B_3 such that, from every point in point in B_0, one can reach every point in B_3 (dashed curves).

The return method has been used to get controllability for many systems, including quantum control systems (see in particular [5]), Schrödinger equations, heat equations, hyperbolic systems as well as many underactuated systems modeled by systems of partial differential equations. For a tutorial presentation of this method, see Jean-Michel's book [24], a book which is a central reference for researchers and graduate students interested in nonlinear control theory. We recommend also the survey article by Glass [35] for a tutorial presentation of various applications of the return method in fluid mechanics.

Quasi-static deformations. The main difficulty to apply the return method is to find non trivial trajectories going from the equilibrium and returning to the equilibrium. In [23] Jean-Michel proposed a method to overcome this difficulty in some cases. The general idea is to first check that there are close to the equilibrium of interest other equilibria which have a controllable linearized control system (they correspond to controls which are different from the one used for the equilibrium point of interest). Then, if these equilibria have suitable stability properties –for suitable feedback laws–, by moving slowly the control one can go from the equilibrium of interest into an arbitrary small neighborhood of these equilibria and then go from a well chosen point close to these equilibria to the equilibrium of interest. This gives trajectories going from the equilibrium and returning to the equilibrium and having a linearized control system. This gives also the local controllability of the nonlinear system at the equilibrium of interest: see Figure 2.

Jean-Michel introduced this method [23] where he in proved the local controllability of a 1-D water tank control system (see Fig. 3). The control is the force F applied to the tank. The motion of the water inside the tank is modeled by means of the Saint-Venant equations (shallow water equations). As a corollary of his result, (small) waves can be destroyed in finite time by applying a suitable (time-varying) force. Note that this cannot be done on the linearized control system: this is the nonlinearity which allows these destructions. Another corollary is that it is possible to move the tank from a position to another one in such a way that at the beginning and at the end the water is at rest. Note that this motion is now possible for the linearized control system. However this does not imply that such a motion is possible for the nonlinear system. Jean-Michel's result shows that such a motion remains possible for the nonlinear system. It is a master piece.

Jean-Michel, in collaboration with E. Trélat, showed in [32] how this strategy can be adapted to get global controllability result for semilinear heat equations.

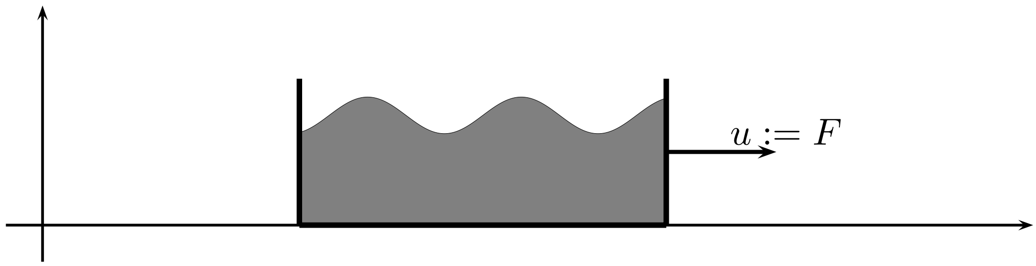

FIGURE 3. A water-tank control system.

Power series expansion. The strategy here is to make a power series expansion of the dynamic around the equilibrium to see if the quadratic, cubic *etc.* terms can give the controllability in the case where the first order expansion (*i.e.* the linearized control system) is not sufficient to get the local controllability. In collaboration with Crépeau, Jean-Michel introduced this method in [26] to prove a new controllability result for a control system, introduced and studied by Rosier in [44], modeled by means of a Korteweg-de Vries equation (the expansion is then made up to the order 3, which requires lengthy computations). It has been used later on for other critical lengths of this Korteweg-de Vries equation [12, 13] and for other control systems (in particular Schrödinger equations, as in [4, 6, 43]).

References

[1] T. Aubin, Équations différentielles non linéaires et problème de Yamabe concernant la courbure scalaire. *J. Math. Pures Appl.* **55** (1976) 269–296.

[2] A. Bahri and J.-M. Coron, On a nonlinear elliptic equation involving the critical Sobolev exponent: the effect of the topology of the domain. *Comm. Pure Appl. Math.* **41** (1988) 253–294.

[3] C. Bardos, G. Lebeau and J. Rauch, Sharp sufficient conditions for the observation, control, and stabilization of waves from the boundary. *SIAM J. Control Optim.* **30** (1992) 1024–1065.

[4] K. Beauchard and J.-M. Coron, Controllability of a quantum particle in a moving potential well. *J. Funct. Anal.* **232** (2006) 328–389.

[5] K. Beauchard, J.-M. Coron and P. Rouchon, Controllability issues for continuous-spectrum systems and ensemble controllability of Bloch equations. *Comm. Math. Phys.* **296** (2010) 525–557.

[6] K. Beauchard and M. Morancey, Local controllability of 1D Schrödinger equations with bilinear control and minimal time. *Math. Control Relat. Fields* **4** (2014) 125–160.

[7] H. Brezis and J.-M. Coron, Multiple solutions of H-systems and Rellich's conjecture. *Comm. Pure Appl. Math.* **37** (1984) 149–187.

[8] H. Brezis and J.-M. Coron, Convergence of solutions of H-systems or how to blow bubbles. *Arch. Rational Mech. Anal.* **89** (1985) 21–56.

[9] H. Brezis, J.-M. Coron and E.H. Lieb, Harmonic maps with defects. *Comm. Math. Phys.* **107** (1986) 649–705.

[10] H. Brézis and L. Nirenberg, Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents. *Comm. Pure Appl. Math.* **36** (1983) 437–477.

[11] R.W. Brockett, Asymptotic stability and feedback stabilization. In *Differential geometric control theory (Houghton, Mich., 1982)*, edited by R.W. Brockett, R.S. Millman and H.J. Sussmann. Vol. 27 of *Progr. Math.* Birkhäuser Boston, Boston, MA (1983) 181–191.

[12] E. Cerpa, Exact controllability of a nonlinear Korteweg-de Vries equation on a critical spatial domain. *SIAM J. Control Optim.* **46** (2007) 877–899.

[13] E. Cerpa and E. Crépeau, Boundary controllability for the nonlinear Korteweg-de Vries equation on any critical domain. *Ann. Inst. Henri Poincaré Anal. Non Linéaire* **26** (2009) 457–475.

[14] J.-M. Coron, Topologie et cas limite des injections de Sobolev. *C. R. Acad. Sci. Paris Sér. I Math.* **299** (1984) 209–212.

[15] J.-M. Coron, Nonuniqueness for the heat flow of harmonic maps. *Ann. Inst. Henri Poincaré Anal. Non Linéaire* **7** (1990) 335–344.

[16] J.-M. Coron, Global asymptotic stabilization for controllable systems without drift. *Math. Control Signals Systems* **5** (1992) 295–312.

[17] J.-M. Coron, Contrôlabilité exacte frontière de l'équation d'Euler des fluides parfaits incompressibles bidimensionnels. *C. R. Acad. Sci. Paris Sér. I Math.* **317** (1993) 271–276.

[18] J.-M. Coron, Linearized control systems and applications to smooth stabilization. *SIAM J. Control Optim.* **32** (1994) 358–386.

[19] J.-M. Coron, On the stabilization of controllable and observable systems by an output feedback law. *Math. Control Signals Systems* **7** (1994) 187–216.

[20] J.-M. Coron, On the stabilization in finite time of locally controllable systems by means of continuous time-varying feedback law. *SIAM J. Control Optim.* **33** (1995) 804–833.

[21] J.-M. Coron, On the controllability of the 2-D incompressible Navier-Stokes equations with the Navier slip boundary conditions. *ESAIM: COCV* **1** (1996) 35–75.

[22] J.-M. Coron, On the controllability of 2-D incompressible perfect fluids. *J. Math. Pures Appl.* **75** (1996) 155–188.

[23] J.-M. Coron, Local controllability of a 1-D tank containing a fluid modeled by the shallow water equations. A tribute to J.L. Lions. *ESAIM: COCV* **8** (2002) 513–554.

[24] J.-M. Coron, *Control and nonlinearity*, Vol. 136 of *Mathematical Surveys and Monographs.* American Mathematical Society, Providence, RI (2007).

[25] J.-M. Coron, G. Bastin and B. d'Andréa Novel, Dissipative boundary conditions for one-dimensional nonlinear hyperbolic systems. *SIAM J. Control Optim.* **47** (2008) 1460–1498.

[26] J.-M. Coron and E. Crépeau, Exact boundary controllability of a nonlinear KdV equation with critical lengths. *J. Eur. Math. Soc. (JEMS)* **6** (2004) 367–398.

[27] J.-M. Coron and A.V. Fursikov, Global exact controllability of the 2D Navier-Stokes equations on a manifold without boundary. *Russian J. Math. Phys.* **4** (1996) 429–448.

[28] J.-M. Coron and J.-M. Ghidaglia, Explosion en temps fini pour le flot des applications harmoniques. *C. R. Acad. Sci. Paris Sér. I Math.* **308** (1989) 339–344.

[29] J.-M. Coron and Q. Lü, Local rapid stabilization for a Korteweg-de Vries equation with a Neumann boundary control on the right. *J. Math. Pures Appl.* **102** (2014) 1080–1120.

[30] J.-M. Coron and Q. Lü, Fredholm transform and local rapid stabilization for a Kuramoto-Sivashinsky equation. *J. Differ. Equ.* **259** (2015) 3683–3729.

[31] J.-M. Coron and H.-M. Nguyen, Dissipative boundary conditions for nonlinear 1-D hyperbolic systems: sharp conditions through an approach via time-delay systems. *SIAM J. Math. Anal.* **47** (2015) 2220–2240.

[32] J.-M. Coron and E. Trélat, Global steady-state controllability of one-dimensional semilinear heat equations. *SIAM J. Control Optim.* **43** (2004) 549–569.

[33] A.V. Fursikov and O.Yu. Imanuvilov, Controllability of evolution equations. Vol. 34 of *Lecture Notes Series*. Seoul National University Research Institute of Mathematics Global Analysis Research Center, Seoul (1996).

[34] O. Glass, Exact boundary controllability of 3-D Euler equation. *ESAIM: COCV* **5** (2000) 1–44.

[35] O. Glass, *La méthode du retour en contrôlabilité et ses applications en mécanique des fluides* [d'après Coron et al.]. *Astérisque*, (348), Exp. No. 1027, vii, 1–16 (2012). Séminaire Bourbaki. Vol. 2010/2011. Exposés 1027–1042.

[36] L.F. Ho, Observabilité frontière de l'équation des ondes. *C. R. Acad. Sci. Paris Sér. I Math.* **302** (1986) 443–446.

[37] O.Yu. Imanuvilov, Boundary controllability of parabolic equations. *Uspekhi Mat. Nauk* **48** (1993) 211–212.

[38] O.Yu. Imanuvilov, Controllability of parabolic equations. *Mat. Sb.* **186** (1995) 109–132.

[39] M. Krstic and A. Smyshlyaev, Boundary control of PDEs. A course on backstepping designs. Vol. 16 of *Advances in Design and Control*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA (2008).

[40] G. Lebeau and L. Robbiano, Contrôle exact de l'équation de la chaleur. *Comm. Partial Differ. Equ.* **20** (1995) 335–356.

[41] J.-L. Lions, Contrôlabilité exacte, perturbations et stabilisation de systèmes distribués. Contrôlabilité exacte. [Exact controllability], With appendices by E. Zuazua, C. Bardos, G. Lebeau and J. Rauch. Tome 1, Vol. 8 of *Recherches en Mathématiques Appliquées [Research in Applied Mathematics]*. Masson, Paris (1988).

[42] J.-L. Lions, Exact controllability, stabilization and perturbations for distributed systems. *SIAM Rev.* **30** (1988) 1–68.

[43] M. Morancey, Simultaneous local exact controllability of 1D bilinear Schrödinger equations. *Ann. Inst. Henri Poincaré Anal. Non Linéaire* **31** (2014) 501–529.

[44] L. Rosier, Exact boundary controllability for the Korteweg-de Vries equation on a bounded domain. *ESAIM: COCV* **2** (1997) 33–55.

[45] D.L. Russell, Nonharmonic Fourier series in the control theory of distributed parameter systems. *J. Math. Anal. Appl.* **18** (1967) 542–560.