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ERRATUM: CYLINDRICAL OPTIMAL REARRANGEMENT PROBLEM LEADING TO A NEW TYPE OBSTACLE PROBLEM

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Abstract. A mistake in the proof of the Theorem 5.5 on page 870 in [H. Mikayelyan, *ESAIM: COCV* **24** (2018) 859–872] has been found.

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The proof of the Theorem 5.5 on page 870 in [1] (below) contains a mistake.

Theorem 1.1. *Let* $\Omega = D \times (0,1)$, $I(u)(x') = \int_0^1 u(x', x_n)\mathrm{d}x_n$, *and* u_1 *and* u_2 *minimize*

$$J(u) = \int_\Omega |\nabla u|^2 + \chi_{\{I(u)>0\}} u \mathrm{d}x,$$

among $W^{1,2}(\Omega)$ *functions with constant boundary data* α_1 *and* α_2 *respectively, where* $0 < \alpha_1 < \alpha_2$. *Then*

$$I(u_1)(x') \le I(u_2)(x'),$$

for $x' \in D$.

In the Step 1 of the proof the following argument has been applied:

For $\delta \ge 0$ *let* u_δ *be the minimizer of the convex functional*

$$J_\delta(u) = \int_\Omega |\nabla u|^2 + \chi_{\{I(u)>\delta\}} u \mathrm{d}x,$$

among the functions $u \in W^{1,2}(\Omega)$ *with boundary values* $u = \alpha_2$. *Let us prove that* $u_2 \le u_\delta$.

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Assume $\tilde{\Omega} = \{x \mid u_2(x) < u_\delta(x)\} \neq \emptyset$ *and set* $u_3 = \min(u_2, u_\delta)$. *If* $\int_{\tilde{\Omega}} |\nabla u_2|^2 \mathrm{d}x < \int_{\tilde{\Omega}} |\nabla u_\delta|^2 \mathrm{d}x$ *then*

$$J_\delta(u_3) < J_\delta(u_\delta). \tag{1.1}$$

Otherwise if $\int_{\tilde{\Omega}} |\nabla u_2|^2 \mathrm{d}x \geq \int_{\tilde{\Omega}} |\nabla u_\delta|^2 \mathrm{d}x$ *then*

$$J(u_3) < J(u_2). \tag{1.2}$$

Equations (1.1) and (1.2) contradict the fact that u_δ *and* u_2 *are minimizers.*

The equation (1.2) is not necessarily true, and the whole proof does not work. If we would "fix" the definition of $\tilde{\Omega}$ to $\tilde{\Omega} = \{x \mid u_2(x) > u_\delta(x)\} \neq \emptyset$, the argument would fail at equation (1.1).

All other results in [1] are unaffected by this and are correct.

It is an open question whether the Theorem 1.1 is correct or not. Some numerical experiments seem to confirm the statement.

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Reference

[1] H. Mikayelyan, Cylindrical optimal rearrangement problem leading to a new type obstacle problem. *ESAIM: COCV* **24** (2018) 859–872.